

Relational Measurement Theory

Chapter 2: Projective Integration

I. INTRODUCTION

Relational Measurement Theory (RMT) is a theory to measure inefficiencies in **derivatives** of extreme complexity. The theory leverages a stable, accurate **primary** as an anchor to map a messy, inaccurate derivative. The **robustness differential** between the derivative and the primary is the foundation of the theory.

The "**Exact Score**" game is a derivative of extreme complexity: participants predict a soccer game final score from 31 outcomes, and any number of options can be purchased per game, creating a near-infinite combination space. The **Money Line** game, by contrast, offers only three outcomes and has proven historically accurate. It serves here as the primary to measure mispricings in the derivative.

First, a profit field uniting the Money Line and Exact Score is constructed under the **Axiom of Locality**. Second, its domain is projected onto a scalar variable called *Key*, preserving differentiability. Finally, **integration** of the transformed function separates differentiable signals from non-differentiable noise.

II. THE PROFIT FIELD AND PROJECTIVE INTEGRATION

1. Odds Space And Profit Field

The **Odds Space** is the 3D space whose axes are the Money Line odds for a Home Win (W), a Draw (D), and a Home Loss (L). A point in the Odds Space represents all games sharing the same Money Line odds. The profit field $p_s(W, D, L)$ is the expected Exact Score profit at point (W, D, L) for a given score outcome s . Each score outcome is treated separately but identically, so the subscript s is omitted for simplicity.

2. Locality And Differentiability

Historical data repeatedly shows that games with similar Money Line odds behave similarly in the Exact Score game, regardless of teams, leagues, or time. The Money Line odds thus act like a continuous background spectrum against which to inspect the Exact Score game.

This is formalized as the **Axiom of Locality**: in Odds Space, proximity between two points implies similar behavior in the Exact Score game. Under this axiom and the following conditions, the profit field becomes differentiable.

- **Domain Interpolation:** Gaps between existing points are filled to create a continuous domain.
- **Infinite Sampling:** Assuming infinitely many games at each point, the expected profit stabilizes and varies smoothly.

3. Projection And Integration

For an ideal smooth field p , the standard approach is to compute its gradient to locate local profit maxima. In reality, however, the domain consists of discrete points, each containing too few games to be statistically significant. The solution is **projective integration** of the profit field, which extracts statistically reliable signals along a chosen direction. This direction is given by a unit vector (w_1, w_2, w_3) . For point (W, D, L) in Odds Space, the scalar Key is defined as (1):

$$Key = w_1 W + w_2 D + w_3 L \quad (1)$$

Geometrically, parallel planes perpendicular to the chosen direction slice the Odds Space, and Key indicates which plane a point falls on. Points on the same plane share the same Key . Key captures only the positional difference along the direction.

Define $q^*(Key)$ as the true expected profit for Key . Geometrically, it is the mean unit area integral across the plane of Key , a construction that preserves the differentiability of p ; Define $q(Key)$ as the mean profit of all sample points at Key . Then

$$\eta_{Key} = q(Key) - q^*(Key)$$

is the sample mean's deviation from the true expectation.

The integration of $q(Key)$ along a dense Key axis is used to separate signal from noise. Let $K_n = \{k_0, k_1, k_2, \dots, k_n\}$ be the ordered set of samples, and let $\Delta k_i = k_i - k_{i-1}$ be the spacing between consecutive Key samples. The Riemann sum $Q(K_n)$ of $q(k)$ over K_n is:

$$Q(K_n) = \sum_{i=1}^n q(k_i) \Delta k_i = \sum_{i=1}^n q^*(k_i) \Delta k_i + \sum_{i=1}^n \eta_i \Delta k_i$$

The Riemann sum decomposes into a signal sum and a noise sum. By the Axiom of Locality, $q^*(k)$ is smooth, so its Riemann sum approximates the integral:

$$\sum_{i=1}^n q^*(k_i) \Delta k_i \approx \int_{k_1}^{k_n} q^*(k) dk$$

For the noise, $\mathbb{E}[\eta_i] = 0$ and the η_i are independent with variance σ^2 . Let $\text{Var}(\eta_i) = \sigma^2$. The expectation and variance of the noise sum are:

$$\mathbb{E} \left[\sum \eta_i \Delta k_i \right] = 0, \quad \text{Var} \left(\sum \eta_i \Delta k_i \right) = \sigma^2 \sum (\Delta k_i)^2$$

Let $\bar{q}_S$ and $\bar{q}_S^*$ be the sample mean and true mean of q over S , respectively. The signal sum becomes:

$$\sum_{i=1}^n q^*(k_i) \Delta k_i = \bar{q}_S^* \sum_{i=1}^n \Delta k_i$$

The SNR for the sum is the ratio of the signal sum to the standard deviation of the noise sum:

$$SNR_{sum} = \frac{\bar{q}_S^* \sum_{i=1}^n \Delta k_i}{\sigma \sqrt{\sum_{i=1}^n (\Delta k_i)^2}} = \frac{\bar{q}_S^*}{\sigma} \sqrt{n} \frac{\text{mean}(\Delta k_i)}{\text{RMS}(\Delta k_i)}$$

$$\text{mean}(\Delta k_i) = \frac{1}{n} \sum_{i=1}^n \Delta k_i, \quad \text{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\Delta k_i)^2}$$

For local slopes:

$$\bar{q}(S) \approx \frac{Q(K_{i+n}) - Q(K_i)}{\Delta K} = \frac{1}{\Delta K} \left(\sum_{j=i+1}^{i+n} q^*(k_j) \Delta k_j + \sum_{j=i+1}^{i+n} \eta_j \Delta k_j \right)$$

The SNR for the slope is identical as sum:

$$SNR_{slope} = \frac{\bar{q}_S^*}{\sigma} \sqrt{n} \frac{\text{mean}(\Delta k_i)}{\text{RMS}(\Delta k_i)}$$

4. Signals

Over a long window S where Q shoots up steadily and smoothly, SNR over S is of great interest. Assume S contains n samples. The sample mean $\bar{q}_S$ and sample variance s_S^2 over S are defined as:

$$\bar{q}_S = \frac{\sum_{i=1}^n q(k_i) \Delta k_i}{\sum_{i=1}^n \Delta k_i}$$

$$s_S^2 = \frac{\sum_{i=1}^n \Delta k_i (q(k_i) - \bar{q}_S)^2}{\sum_{i=1}^n \Delta k_i}$$

The empirical $\widehat{SNR}$ (applicable to both the cumulative sum and the local slope) is then:

$$\widehat{SNR} = \frac{\bar{q}_S}{s_S} \sqrt{n} \frac{\text{mean}(\Delta k_i)}{\text{RMS}(\Delta k_i)}$$